

Interpretations of Cosmological Spectral Shifts

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Abstract

It is shown that for Robertson-Walker models with flat or closed space sections, *all* of the cosmological spectral shift can be attributed to a curved connection (and thus indirectly to space-time curvature). For Robertson-Walker models with hyperbolic space sections, it is shown that cosmological spectral shifts uniquely split up into “kinematic” and “gravitational” parts provided that distances are small. For large distances no such unique split-up exists in general. A number of common, but incorrect assertions found in the literature regarding interpretations of cosmological spectral shifts, is pointed out.

1 Introduction

Although there is in general no dispute about actual predictions coming from universe models based on General Relativity (GR), *interpretations* of the nature of the cosmic expansion/contraction and cosmic spectral shifts predicted by these models, have on the other hand been subject to some lengthy controversy. (See e.g., [1] and references therein.) The controversial question is, when pulses of electromagnetic radiation are emitted and received between “fundamental observers” (FOs) following the cosmic fluid (with no peculiar motions); what is the nature of the resulting spectral shifts?

What is completely uncontroversial is the fact that the ratio of the observed and emitted wavelengths λ_{obs} and λ_{em} , respectively, is related to the ratio of the cosmic scale factors at observation and emission a_{obs} and a_{em} , respectively, and the cosmological spectral shift z via the formula $\lambda_{\text{obs}}/\lambda_{\text{em}} = a_{\text{obs}}/a_{\text{em}} = 1 + z$. However, the controversial part of said question is to what extent, if any, such cosmological spectral shifts can be interpreted as Doppler shifts in flat space-time.

One school of thought claims that, since the Equivalence Principle (EP) says that space-time is locally flat, the nature of cosmological spectral shifts must be interpreted as Doppler shifts in flat space-time in the limit where distances between FOs go to zero. The opposite view is that, since the FOs are at rest with respect to the cosmic fluid (defining an “expanding frame”) and since cosmological spectral shifts are given

from the formula shown above rather than from the special-relativistic Doppler formula, cosmological spectral shifts should in general have nothing to do with Doppler shifts, even for arbitrarily small distances. These interpretations are known as the “kinematic” (in the narrow sense of the word) and the “expanding space” interpretations, respectively. For curved space-times, it is often not made clear if the “expanding space” interpretation attributes cosmological spectral shifts entirely to space-time curvature. However, some authors (incorrectly) claim this and thus that cosmological spectral shifts should be interpreted as some sort of “gravitational” spectral shifts whenever space-time is not flat.

At first glance, at least for small distances, the difference between these interpretations seems to be the rather trivial matter of describing the same physics using different frames. Thus, to some people it would seem reasonable to proclaim both interpretations valid for small distances and just representing equivalent points of view. However, before such a solution is endorsed, it must be established that the various interpretations are mathematically consistent. But it turns out that they aren’t, since interpretations may be associated with geometrical restrictions. In particular, in this paper we show that the “kinematic” interpretation is in general mathematically inconsistent with the geometry of the Robertson-Walker (RW) models, so that this interpretation is not valid generally. On the other hand, we show that what is crucial for interpreting spectral shifts in the RW-models is not the mere existence of an “expanding frame”, but rather how this frame relates to space-time curvature. And as we shall see in what follows, this relationship differs between types of RW-models. That is why the “expanding space” interpretation is not useful in general, either.

2 “Kinematic” and “gravitational” spectral shifts

To understand what is actually meant by “kinematic” and “gravitational” spectral shifts in context of the RW-models, it is necessary to define these concepts mathematically. Such definitions should be formulated together with a recipe for spectral shift split-up into “kinematic” and “gravitational” parts. It would perhaps seem natural to insist that said definitions must be based on a general spectral shift split-up coming from some geometrical procedure being valid for all RW-models. However, it is shown in this section that such an approach cannot be justified if the definition of “kinematic” spectral shift is required to be based on the definition of spectral shifts in Special Relativity (SR). Abandoning this requirement is certainly possible, but then the definitions of “kinematic” and “gravitational” spectral shifts will be only formal and not useful for interpretations.

The mathematical framework considered in this paper is given by the usual 4-dimensional

semi-Riemannian manifold $(\mathcal{M}, \mathbf{g})$. In addition it is required that (at least a subset of) $(\mathcal{M}, \mathbf{g})$ can be foliated into a continuous sequence of 3-dimensional spatial hypersurfaces $\mathcal{S}(x^0)$ parameterized by a time function x^0 . The *fundamental observers* are “preferred” observers defined from the foliation by the criterion that their world lines are everywhere continuous and orthogonal to $\mathcal{S}(x^0)$. The choice of foliation (and thus of time coordinate) is required to be unambiguously made from purely geometrical selection criteria.

Since this paper is about the RW-manifolds, the analysis presented here is restricted to one specific choice of selection criteria. This specific choice of selection criteria picks out space-time manifolds that can be foliated into a set of hypersurfaces such that the spatial geometry is everywhere isotropic and homogeneous. Moreover, the unit normal vector field to the hypersurfaces should not be a (timelike) Killing vector field. This last criterion excludes static manifolds with topology $\mathcal{S} \times \mathbf{R}$ equipped with a foliation determined from the product topology (here \mathcal{S} is one of the space geometries \mathbf{R}^3 , \mathbf{S}^3 or \mathbf{H}^3). These selection criteria uniquely yield the RW-manifolds each equipped with the specified “preferred” foliation, determining “preferred” hypersurfaces. (In the next section, we will also consider other foliations of “open” RW-manifolds than the “preferred” ones as useful for specific calculations.)

Given $(\mathcal{M}, \mathbf{g})$ and a foliation of it into spatial hypersurfaces $\mathcal{S}(x^0)$, spectral shifts obtained by exchanging light signals between nearby FOs are unambiguously determined from the space-time geometry. Moreover, this holds irrespectively of any particular choice of field equations, so it is not necessary to assume the validity of the GR field equations. Consequently, the results obtained in this paper depend only on the geometry of space-time with no extra assumptions. In particular, no particular relationship between geometrical quantities and matter sources is assumed to hold. Nevertheless, we will by convention call spectral shifts entirely due to space-time curvature for “gravitational” spectral shifts. (See Definition 1 below for such a situation.)

To calculate spectral shifts in general, there exists a simple geometric procedure, as first pointed out by Synge. That is, imagine a pulse of electromagnetic radiation being emitted at some given event and subsequently observed at some other given event. Then, by parallel-transporting the 4-velocity \mathbf{u}_e of the emitter along the null curve connecting the given events, the parallel-transported 4-velocity of the emitter can be projected into the local rest frame of the observer. This yields a 3-velocity that can be inserted into the special-relativistic Doppler formula to give the desired spectral shift. (For the full mathematical details of this procedure, see [2].) This procedure works for any relativistic space-time (and even for cases where the space-time geometry is not semi-Riemannian [3]), and implies that it is always possible to interpret spectral shifts as due to the Doppler

effect in *curved* space-time, without any geometrical restrictions whatsoever. Moreover, this procedure illustrates that what is relevant for calculating spectral shifts are the connection coefficients, since these enter into the mathematical expression for parallel-transport. Any non-zero values of the connection coefficients may arise due to the choice of coordinates, a curved connection, or both.

For the RW-manifolds, the connection coefficients relevant for spectral shifts obtained from photon signalling between FOs, are uniquely determined from the evolution with time of the spatial geometry \mathbf{h} of the “preferred” foliation in the direction of the unit vector field \mathbf{n} normal to the hypersurfaces. That is, the relevant quantity is given from the *extrinsic curvature tensor* \mathbf{K} defined by (in component notation using a general coordinate system $\{x^\mu\}$ and using Einstein’s summation convention, see, e.g., ref. [4], p. 256)

$$K_{\mu\nu} \equiv \frac{1}{2} \mathcal{L}_{\mathbf{n}} h_{\mu\nu} = \frac{1}{2} \left(h_{\mu\nu, \alpha} n^\alpha + h_{\alpha\nu} n^\alpha{}_{, \mu} + h_{\mu\alpha} n^\alpha{}_{, \nu} \right), \quad (1)$$

where $\mathcal{L}_{\mathbf{n}}$ denotes the Lie derivative in the \mathbf{n} -direction and where a comma denotes a partial derivative. Using a spherically symmetric hypersurface-orthogonal coordinate system $\{x^0, \chi, \theta, \phi\}$ where $n^\mu = (1, 0, 0, 0)$, the metrics of the RW-manifolds (equipped with the “preferred” foliation) take the form

$$ds^2 = -(dx^0)^2 + a^2(x^0) \left(d\chi^2 + \Sigma^2(\chi) d\Omega^2 \right), \quad d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2, \quad (2)$$

where $a(x^0)$ is the scale factor of the hypersurfaces and where

$$\Sigma(\chi) = \begin{cases} \sin\chi & \text{for hypersurfaces with spherical geometry,} \\ \chi & \text{for hypersurfaces with flat geometry,} \\ \sinh\chi & \text{for hypersurfaces with hyperbolical geometry.} \end{cases} \quad (3)$$

Using the form (2) of the metric, equation (1) for the extrinsic curvature of the “preferred” hypersurfaces (embedded into the RW-manifolds) takes the form (with $\dot{a} \equiv \frac{da}{dx^0}$)

$$K_{\mu\nu} = \frac{1}{2} \frac{\partial}{\partial x^0} h_{\mu\nu} = \frac{1}{c} H(x^0) h_{\mu\nu}, \quad H(x^0) \equiv \frac{\dot{a}}{a}, \quad (4)$$

where $H(x^0)$ is the Hubble parameter. In a hypersurface-orthogonal coordinate system (such as used in equation (2)), the nonzero components of the spatial metric \mathbf{h} can be found directly from the spatial part of the space-time metric \mathbf{g} , yielding the nonzero components of \mathbf{K} from equation (4). Note that \mathbf{K} is a tensor field on space (since the scalar product $\mathbf{K} \cdot \mathbf{n} = \mathbf{0}$), and that $H(x^0) = \frac{c}{3} K^\mu{}_\mu$ is a scalar field (constructed from the “preferred” foliation).

Now it turns out that there exists an expression for the intrinsic Riemann curvature tensor \mathbf{P} of the hypersurfaces in terms of the space-time Riemann curvature tensor \mathbf{R} and the extrinsic curvature tensor \mathbf{K} . This is the well-known Gauss equation (see, e.g., ref. [4], p. 258), and in component notation it reads

$$P^\alpha_{\beta\gamma\delta} = R^\lambda_{\rho\mu\nu} h^\alpha_\lambda h^\rho_\beta h^\mu_\gamma h^\nu_\delta + K^\alpha_\delta K_{\beta\gamma} - K^\alpha_\gamma K_{\beta\delta}. \quad (5)$$

Moreover, contracting equation (5) twice and using equation (4), we get

$$P = R + 2R_{\alpha\beta}n^\alpha n^\beta - 6H^2/c^2, \quad \Rightarrow \quad H^2 = \frac{c^2}{6} [2G_{\alpha\beta}n^\alpha n^\beta - P], \quad (6)$$

where R and P are the scalar curvatures of space-time and space, respectively, and where $R_{\alpha\beta}$ and $G_{\alpha\beta}$ are the components of the Ricci tensor and the Einstein tensor on space-time, respectively.

Now we see from equation (4) that there can be no spectral shift (detected by photon signalling between FOs) if $\mathbf{K} = \mathbf{0}$. Therefore, to make sense of any “kinematic” part of the spectral shift having a similarity with spectral shifts in SR, it must be possible to have a limit where the relevant part of \mathbf{R} may be neglected but such that $\mathbf{K} \neq \mathbf{0}$. If such a limit does not exist, the spectral shift must be entirely due to space-time curvature (i.e., “gravitational”). Whether or not such a limit exists can be found from equation (6). (The Weyl tensor vanishes identically for the RW-manifolds, so the Ricci tensor (or the Einstein tensor) captures all aspects of space-time curvature.) We thus have the definition

Definition 1 *Assume as given a semi-Riemannian manifold $(\mathcal{M}, \mathbf{g})$ of RW-type and a foliation of it into “preferred” isotropic and homogeneous spatial hypersurfaces $\mathcal{S}(x^0)$ (with unit normal vector field \mathbf{n}) defined from equation (2). Also denote any hypersurface metric by \mathbf{h} with extrinsic curvature tensor \mathbf{K} , intrinsic Riemann curvature tensor \mathbf{P} and intrinsic curvature scalar P . The space-time Einstein curvature tensor is denoted by \mathbf{G} . Then, if it is not possible to have $G_{\alpha\beta}n^\alpha n^\beta$ arbitrary small independent of P with $\mathbf{K} \neq \mathbf{0}$, any spectral shift obtained by photon signalling between FOs is entirely due to space-time curvature.*

If a situation like that described in Definition 1 occurs, any definitions and spectral splits that allow for a non-zero “kinematic” spectral shift, do not make sense. This is why the approach of starting with general definitions of “kinematic” and “gravitational” spectral shifts valid for any RW-models cannot be justified, since, as we shall see, the situation described in Definition 1 occurs for all RW-models where the “preferred” foliation consists of flat or spherical hypersurfaces.

To prove that the situation described in Definition 1 occurs for the case of flat hyper-surfaces, it is obvious from equation (6) that it is not possible to have a flat RW-manifold with flat spatial sections (i.e., $\mathbf{P} \equiv \mathbf{0}$) and still have $H(x^0) \neq 0$. That is, the requirements $P = 0$, $\mathbf{G} = \mathbf{0}$ mean that equation (6) is satisfied only for $H(x^0) = 0$. Note that this is not in any way a coordinate-dependent result. Thus we arrive at the conclusion that to have a RW-manifold with flat space sections and at the same time $H(x^0) \neq 0$, space-time must be curved. This means that according to Definition 1, *spectral shifts observed by exchanging photons between FOs in a RW-manifold with flat spatial sections are entirely due to space-time curvature*. Since this result holds irrespective of distances between FOs, we are forced to interpret the relevant spectral shifts as purely “gravitational” for all RW-manifolds with flat spatial sections.

A similar result holds for the closed RW-manifolds (with spherical spatial sections). In this case $P = \frac{6}{a^2} > 0$, and equation (6) yields that it is not possible to have $G_{\alpha\beta}n^\alpha n^\beta$ arbitrary small independent of P such that $H^2 > 0$. This result is a consequence of the fact that it is not possible to foliate Minkowski space-time into hypersurfaces with \mathbf{S}^3 -geometry. So, from Definition 1 we have that *spectral shifts observed by exchanging photons between FOs in a RW-manifold with closed (spherical) spatial sections are entirely due to space-time curvature*. We are then forced to interpret all relevant spectral shifts as purely “gravitational” for all RW-manifolds with spherical spatial sections as well.

We are thus left with open RW-manifolds foliated into hyperbolical hypersurfaces as the only nontrivial case when it comes to interpretations. In this case $P = -\frac{6}{a^2} < 0$, so it is indeed possible to choose $G_{\alpha\beta}n^\alpha n^\beta$ arbitrary small independent of P in equation (6) together with $H^2 > 0$ (i.e., $\mathbf{K} \neq \mathbf{0}$), so that the situation described in Definition 1 does not occur. This means that for the case $P < 0$, it may make sense to define a spectral shift split-up into “kinematic” and “gravitational” parts. In particular it is possible to choose $\mathbf{G} = \mathbf{0}$, $P < 0$ in equation (6) together with $H^2(x^0) > 0$, since (part of) Minkowski space-time can be foliated into hypersurfaces with \mathbf{H}^3 -geometry. This special case is the “empty” RW space-time (Milne model), which is just a subset of Minkowski space-time and thus flat. The line element is given by equations (2) and (3) by setting $a(x^0) = x^0$, i.e.,

$$ds^2 = -(dx^0)^2 + (x^0)^2(d\chi^2 + \sinh^2\chi d\Omega^2). \quad (7)$$

In this case it is obvious that the “kinematic” interpretation is correct since the cosmic expansion is entirely due to the “preferred” choice of space-time foliation into space and time. That is, by switching to standard coordinates $r \equiv x^0 \sinh\chi$, $x^{0'} \equiv x^0 \cosh\chi$, another foliation is chosen and the line element takes the familiar Minkowskian form expressed

in spherical coordinates. This means that, by performing a suitable coordinate transformation, it is possible to eliminate the connection coefficients altogether. Moreover, the cosmic redshift can be found *locally* from the speed $w_{\mathcal{F}}$ of a FO relative to a local observer moving normal to the $x^{0'} = \text{constant}$ hypersurfaces. We will exploit this fact when treating general open models in section 3.

We may now define a “purely kinematic” spectral shift as one occurring in a RW-manifold foliated into hyperbolic hypersurfaces for situations where the difference between a curved and a flat connection does not matter for photon propagation between nearby FOs. That is, it may be possible that the contribution to equation (5) from extrinsic curvature at some epoch x_0^0 is identical to the contribution to equation (5) from extrinsic curvature of a hyperbolic hypersurface with identical geometry but embedded in Minkowski space-time. (In such a situation, H^2 and P will be identical for the two hypersurfaces, meaning that $G_{\alpha\beta}n^\alpha n^\beta$ must vanish even at the hypersurface embedded in curved space-time in order not to violate equation (6).) To find what the latter contribution is, it is convenient to use a hypersurface-orthogonal coordinate system as used in equation (2). One then finds that the contribution to equation (5) from extrinsic curvature depends on $\dot{a}^2(x^0)$ but not on $a(x^0)$. Since $\dot{a}^2(x^0) = 1$ for the Milne model given in equation (7), we have the definition:

Definition 2 *Assume as given a semi-Riemannian manifold $(\mathcal{M}, \mathbf{g})$ of RW-type and a foliation of it into isotropic and homogeneous spatial hypersurfaces $\mathcal{S}(x^0)$ with hyperbolic intrinsic geometry (see equation (2)). Also assume the existence of some hypersurface $\mathcal{S}(x_0^0)$ with spatial metric $\mathbf{h}(x_0^0)$ and extrinsic curvature tensor $\mathbf{K}(x_0^0)$ given from equation (4) (in a hypersurface-orthogonal coordinate system) with $|\dot{a}(x_0^0)| = 1$. Then spectral shifts resulting from photon signalling between nearby FOs close to $\mathcal{S}(x_0^0)$ are defined to be “purely kinematic” in the limit where distances between FOs go to zero and $x^0 \rightarrow x_0^0$.*

Definition 2 is based on the fact that for the open RW-models, it may be possible to have a situation where the contribution to equation (5) from extrinsic curvature at the epoch x_0^0 is identical to that of a hyperbolic hypersurface with identical geometry in the Milne model at epoch $x_0^0 + b$ where b is some constant. In such a situation, the relevant connection coefficients for the open model at epoch x_0^0 will be identical to those for the Milne model at epoch \tilde{x}_0^0 , where the scale factor is given by $a(\tilde{x}^0) = \tilde{x}^0 = x^0 + b$ and such that $a(\tilde{x}_0^0) = x_0^0 + b = a(x_0^0)$. In the next section, we will give some specific examples of open RW-models where this situation occurs.

3 Spectral shift split-up

The main result of the previous section was that *all* RW-models foliated into flat or spherical hypersurfaces satisfy the situation described in Definition 1. Therefore, *all* the cosmic spectral shift in these models must be due to space-time curvature, so that a spectral shift split-up into “kinematic” and “gravitational” parts does not make sense for these RW-models. On the other hand, we show in this section that for RW-models foliated into hyperbolic hypersurfaces, such a spectral shift split-up can be defined consistently for small distances and in agreement with Definition 2.

To define a split-up of spectral shifts into “kinematic” and “gravitational” parts (valid for RW-models with hyperbolic spatial sections), it is convenient to change the space-time foliation. Note that the change of foliation is made because it makes calculations easier and the correspondence with the Milne model clearer. Note in particular that the FOs are still being defined as those observers moving normal to the “preferred” foliation given from equation (2), and that the defined spectral shift split-up applies only to the FOs. Observers moving normal to the new foliation only play an auxiliary role.

The relevant change of foliation is made by switching to new coordinates $r \equiv a(x^0) \sinh \chi$, $x^{0'} \equiv a(x^0) \cosh \chi$, so that the line element (2) transforms to

$$\begin{aligned}
 ds^2 = & - \left(\frac{\dot{a}^{-2} - \frac{r^2}{(x^{0'})^2}}{1 - \frac{r^2}{(x^{0'})^2}} \right) (dx^{0'})^2 - 2 \frac{r}{x^{0'}} \left(\frac{1 - \dot{a}^{-2}}{1 - \frac{r^2}{(x^{0'})^2}} \right) dx^{0'} dr \\
 & + \left(\frac{1 - \frac{r^2}{(x^{0'})^2} \dot{a}^{-2}}{1 - \frac{r^2}{(x^{0'})^2}} \right) dr^2 + r^2 d\Omega^2, \quad \frac{r}{x^{0'}} < \inf\{|\dot{a}|^{-1}, |\dot{a}|\}, \quad (8)
 \end{aligned}$$

where $\dot{a} \equiv \frac{da}{dx^0} = \dot{a}(x^{0'}, r)$ is now a function of both the time coordinate and the radial coordinate. We can now use equation (8) to find the spectral shift of light emitted by a FO located at the radial coordinate χ (i.e., with coordinate motion $\frac{dr}{dx^{0'}} = \frac{r}{x^{0'}}$) as observed by a FO located at the origin $\chi = 0$. Moreover, for small values of χ , we will show that to lowest order in $\frac{r}{x^{0'}}$, this spectral shift can be written as a sum of “kinematic” and “gravitational” contributions. Note that, since $a(x^0) = \sqrt{(x^{0'})^2 - r^2} = x^{0'} + O(2)$, the choice of foliation (i.e., the choice of time coordinate) leading to equation (8) is unique to first order in the small quantity $\frac{r}{x^{0'}}$, but not higher. This means that any split-up of spectral shifts into “kinematic” and “gravitational” parts is limited to small distances.

To arrive at the desired spectral shift split-up, we first split up the 4-velocity $\mathbf{u}_{\mathcal{F}}$ of the FOs into parts normal and tangential to the hypersurfaces $x^{0'} = \text{constant}$. This split-up

reads

$$\mathbf{u}_{\mathcal{F}} = \gamma(c\tilde{\mathbf{n}} + \mathbf{w}_{\mathcal{F}}), \quad \gamma \equiv (1 - \frac{w_{\mathcal{F}}^2}{c^2})^{-1/2}, \quad (9)$$

where $\tilde{\mathbf{n}}$ is the unit normal vector field of the hypersurfaces $x^{0'} = \text{constant}$ and $\mathbf{w}_{\mathcal{F}}$ is the 3-velocity difference (with squared norm $w_{\mathcal{F}}^2$) between a FO and a local observer moving normal to these hypersurfaces. Only the r -component of this equation is of interest, and it reads

$$w_{\mathcal{F}}^r = \frac{dr}{dx^{0'}} \frac{dx^{0'}}{d\tau_N} + \frac{N^r}{N}c, \quad cd\tau_N = Ndx^{0'} = \sqrt{\frac{1 - \frac{r^2}{(x^{0'})^2}}{\dot{a}^2 - \frac{r^2}{(x^{0'})^2}}}dx^{0'}, \quad N^r = \frac{r}{x^{0'}} \frac{1 - \dot{a}^2}{[\dot{a}^2 - \frac{r^2}{(x^{0'})^2}]}, \quad (10)$$

where N is the lapse function and N^r is the shift vector r -component of observers moving normal to the hypersurfaces $x^{0'} = \text{constant}$. The relation of these quantities to the line element given by equation (8) can be found from the formula

$$ds^2 = [N^r N_r - N^2](dx^{0'})^2 + 2N_r dx^{0'} dr + \tilde{h}_{ij} dx^i dx^j, \quad (11)$$

where \tilde{h}_{ij} are the components of the hypersurface metric, found explicitly from equation (8). It is straightforward to calculate the speed $w_{\mathcal{F}}$, and we find that

$$w_{\mathcal{F}} \equiv \sqrt{w_{\mathcal{F}}^i w_{\mathcal{F}}^j \tilde{h}_{ij}} = \frac{\tanh \chi}{|\dot{a}|}c = \frac{r}{|\dot{a}|x^{0'}}c. \quad (12)$$

The speed $w_{\mathcal{F}}$ can now be put into the special-relativistic Doppler formula to find the spectral shift as measured by a local observer moving normal to the hypersurfaces $x^{0'} = \text{constant}$. Applied to the Milne model, this approach yields a local determination of the cosmological spectral shift in flat space-time. It is thus natural to define a more general “kinematic” spectral shift z_k valid for open models, found locally and given by

$$1 + z_k \equiv \sqrt{\frac{1 \pm w_{\mathcal{F}}/c}{1 \mp w_{\mathcal{F}}/c}}, \quad \Rightarrow \quad z_k = \pm \frac{r}{|\dot{a}|x^{0'}} + O(2) = \dot{a}^{-2} \frac{H(x^0)r}{c} + O(2). \quad (13)$$

We see that if $|\dot{a}| \rightarrow 1$, we have a situation where the spectral shift is defined as “purely kinematic” according to Definition 2, and is identical to the special-relativistic result. Moreover, one may easily see that this definition yields the expected result $z_k = 0$ if extrapolated to RW-models with flat spatial sections. That is, the coordinate transformation $r \equiv a(x^0)\chi$, $x^{0'} \equiv a(x^0)$ yields the counterpart expression to equation (8) of the line element valid for RW-models with flat space sections. Hence $w_{\mathcal{F}} \equiv 0$ due to the fact that this coordinate transformation does not yield a new foliation so that the FOs move orthogonally to the $x^{0'} = \text{constant}$ hypersurfaces as well.

Next, we note that an observer moving with constant r -coordinate and a local observer moving normal to the hypersurfaces $x^{0'} = \text{constant}$ will not have coinciding world lines, but will have a 3-velocity difference \mathbf{w} . We will now show that the corresponding speed w can be used to define a local determination of “gravitational” spectral shift. To do that, similar to equations (10) and (12), we find the quantities

$$w^r = \frac{N^r}{N}c = \frac{rc}{x^{0'}} \frac{1 - \dot{a}^2}{\sqrt{(\dot{a}^2 - \frac{r^2}{(x^{0'})^2})(1 - \frac{r^2}{(x^{0'})^2})}}, \quad \Rightarrow \quad w = \frac{|1 - \dot{a}^2|r}{|\dot{a}|x^{0'}(1 - \frac{r^2}{(x^{0'})^2})}c, \quad (14)$$

and these expressions vanish in the limit $|\dot{a}| \rightarrow 1$, as they should for “gravitational” quantities. It is thus natural to associate the corresponding spectral shift with space-time curvature, i.e., it should be due to “gravitational” causes. Since the sign of w^r depends on whether $1 - \dot{a}^2$ is positive or negative, the contribution to the spectral shift with respect to the emitting FO will be either negative or positive, respectively. That is, what enters into the special-relativistic Doppler formula is not the speed w , but rather the quantity w_{\pm} defined by

$$w_{\pm} \equiv \frac{(\dot{a}^2 - 1)r}{|\dot{a}|x^{0'}(1 - \frac{r^2}{(x^{0'})^2})}c, \quad (15)$$

which may be used to define a “gravitational” spectral shift z_g (valid for open models) given by

$$1 + z_g \equiv \sqrt{\frac{1 \pm w_{\pm}/c}{1 \mp w_{\pm}/c}}, \quad \Rightarrow \quad z_g = \pm \frac{r(\dot{a}^2 - 1)}{|\dot{a}|x^{0'}} + O(2). \quad (16)$$

Again, one may easily check that a similar definition extrapolated to RW-models with flat spatial sections yields the expected lowest-order result $z_g = \frac{H(x^0)r}{c}$.

If the observer moving with constant r -coordinate emits light that is detected by the FO residing in the spatial origin, the resulting spectral shift will be of higher order in the small quantity $\frac{r}{x^{0'}}$, so this contribution can be neglected. (Here, a possible effect of nonzero $\ddot{a} \equiv \frac{d^2 a}{dx^{0'^2}}$ may also be neglected if said small quantity is small enough.) This means that to lowest order, the total spectral shift measured by the FO residing at the origin can be written as a sum of “kinematic” and “gravitational” contributions, and that this spectral shift is given by

$$\begin{aligned} z &= z_k + z_g + O(2) = \pm \left[(\dot{a}^2 - 1) + 1 \right] \frac{r}{|\dot{a}|x^{0'}} + O(2) \\ &= \pm |\dot{a}| \frac{r}{x^{0'}} + O(2) = \frac{H(x^0)r}{c} + O(2), \end{aligned} \quad (17)$$

which is the familiar lowest-order expression for cosmological spectral shifts. Moreover, for small distances the split-up defined in equation (17) is unique. On the other hand, for large distances, cosmological spectral shifts in an open RW-model cannot uniquely be split up into “kinematic” and “gravitational” parts. This is so since other foliations (coinciding with the foliation defined by the $x^{0'}$ -coordinate for small distances but differing from it for large distances) may be equally well be used when defining spectral shift split-up by the method described above.

To illustrate the meaning of the split-up defined in equation (17), we finish this section with some simple examples. First, we choose a form of the scale factor consistent with a radiation-dominated universe as predicted by GR, i.e.,

$$a(x^0) = \sqrt{a^* x^0}, \quad \dot{a} = \frac{1}{2} \sqrt{\frac{a^*}{x^0}} = \frac{a^*}{2a}, \quad z_k = \frac{2r}{a^*} + O(2), \quad z_g = \left[\frac{1}{2x^0} - \frac{2}{a^*} \right] r + O(2), \quad (18)$$

where a^* represents an arbitrary constant reference scale. We note that z_k does not depend on epoch. Furthermore, we see that z_g is positive for early epochs, vanishes for $x^0 = a^*/4$, and becomes negative for later epochs. The particular epoch where z_g vanishes is (of course) determined by the condition $\dot{a} = 1$. At this epoch, the expansion of the universe momentarily mimics that of the Milne model with a “shifted” scale factor given by $a(x^0) = x^0 + a^*/4$. Hence, since we can neglect the effect of \ddot{a} for small enough distances, in this limit the “kinematic” interpretation of the cosmic redshift will hold, despite the fact that space-time is not flat. However, at earlier epochs $\dot{a} > 1$ so the universe expands faster than an “empty” universe, yielding an extra redshift. This means that gravity has not had enough time to slow down the expansion sufficiently over the universe’s history. Similarly, for later epochs, $\dot{a} < 1$ and the universe expands slower than an “empty” universe, giving an extra blueshift. Gravity has then had enough time to slow down the expansion sufficiently so that it expands slower than in the Milne model.

Second, choose a form of the scale factor consistent with a matter-dominated universe as predicted by GR, i.e.,

$$a(x^0) = (a^*)^{\frac{1}{3}} (x^0)^{\frac{2}{3}}, \quad \Rightarrow \quad \dot{a} = \frac{2}{3} \left(\frac{a^*}{x^0} \right)^{\frac{1}{3}} = \frac{2}{3} \sqrt{\frac{a^*}{a}},$$

$$z_k = \frac{3r}{2(a^*)^{\frac{2}{3}} (x^0)^{\frac{1}{3}}} + O(2), \quad z_g = \left[\frac{2}{3x^0} - \frac{3}{2(a^*)^{\frac{2}{3}} (x^0)^{\frac{1}{3}}} \right] r + O(2), \quad (19)$$

where again a^* is an arbitrary constant reference scale. We note that, unlike the previous example, in this case z_k depends on epoch. Moreover, z_g vanishes for the epoch $x^0 = \frac{8}{27} a^*$, and at this epoch the universe expands momentarily as an “empty” RW-model with a “shifted” scale factor given by $a(x^0) = x^0 + \frac{4}{27} a^*$. So at this particular epoch the cosmic

redshift should be interpreted as a pure “kinematic” effect in flat space-time for small distances (even though space-time is not flat). However, this interpretation breaks down for other epochs.

A final example is given where the scale factor is determined by a (positive) cosmological constant Λ , i.e.,

$$a(x^0) = \sqrt{\frac{3}{\Lambda}} \sinh \left[\sqrt{\frac{\Lambda}{3}} x^0 \right], \quad \Rightarrow \quad \dot{a} = \cosh \left[\sqrt{\frac{\Lambda}{3}} x^0 \right] = \sqrt{1 + \frac{\Lambda}{3} a^2},$$

$$z_k = \frac{r}{a \sqrt{1 + \frac{\Lambda}{3} a^2}} + O(2), \quad z_g = \frac{\Lambda}{3} \frac{ar}{\sqrt{1 + \frac{\Lambda}{3} a^2}} + O(2). \quad (20)$$

We note that in this case, for very early epochs $x^0 \rightarrow 0$, the cosmic expansion mimics that of the Milne model so that $z_k \rightarrow \infty$ and $z_g \rightarrow 0$ in this limit. However, at late epochs z_k decreases exponentially, so it can soon be neglected. Thus, at late epochs, the cosmic redshift should be interpreted as due to space-time curvature (i.e., “gravitational”) with negligible “kinematic” contribution.

4 Fallacies of popular cosmology

The results obtained in section 2 for the flat and closed RW-models were also arrived at by Roukema [1], using topological methods. That is, by changing the topology of the spatial sections of the relevant metrics (given by equations (2) and (3)) from simply connected to multiply connected, but without changing the geometry, it was shown that a contradiction arises if spectral shifts are interpreted as due to the Doppler effect in flat space-time. On the other hand, considering a less general case than for flat and closed RW-models, this contradiction did not occur for open RW-models, except for certain large distances. This means that the results obtained in section 3 do not match the corresponding results in [1], so there seems to be a contradiction. (This would indicate that using topological methods is not sufficient for analysing the RW-models with hyperbolic spatial sections.) On the other hand, searching the relevant literature, one finds that reference [1] is about the only one emphasizing the crucial role of the spatial geometry when it comes to interpretations. Otherwise, what has been discussed is the “kinematic” versus the “expanding space” views with no due weight on spatial geometry. It has even been claimed [5] that spatial geometry is irrelevant for interpretations of certain *gedanken*-experiments involving radar distances and spectral shifts, since the calculated results of such hypothetical experiments do not depend on the spatial parts of the metrics (2). But this argument is flawed since interpretations are properties of

models rather than of experimental results. There is absolutely no scientific requirement that different interpretations should be experimentally distinguishable.

Another, common but incorrect assertion is that the effects on spectral shifts of curved space-time, as compared to “kinematic” effects, can always be neglected in the RW-models for sufficiently small distances. The argument is that, since one may always choose local coordinates such that the tangent space-time at some event \mathcal{P} (given, e.g., by $x^0 = x_0^0, \chi = 0$) takes the standard Minkowskian form, and in a (small) neighbourhood of \mathcal{P} approximates the space-time metric to first order in small quantities, the effects of space-time curvature can be made negligible in a sufficiently small neighbourhood of \mathcal{P} . (The EP ensures that such a coordinate system can be found for *any* metric.) In such a coordinate system, the FOs will have a (radial) velocity field $v(r) = H_0 r + O(2)$ (where H_0 is the local Hubble parameter) as seen from \mathcal{P} . It is then argued that $v(r)$ should be interpreted as a purely “kinematic” velocity field in flat space-time. However, one problem with this argument is that the constant H_0 is in effect equal to the connection coefficient $\Gamma_{0\chi}^\chi = \Gamma_{\chi 0}^\chi$ (evaluated at the event \mathcal{P}) obtained from the in general *curved* metric (2). This means that, by performing this procedure one has actually transformed *all* relevant effects, “kinematic” and curvature effects alike, into $v(r)$. In other words, since nothing at all is said regarding the nature of $\Gamma_{0\chi}^\chi$, said procedure is in fact *irrelevant* for interpretations of the expansion.

To see that the effects of a curved connection cannot be neglected in general, even for small distances, it is illustrating to write the scale factor $a(x^0)$ as a Taylor series around the event \mathcal{P} , i.e.,

$$a(x^0) = a(x_0^0) + \dot{a}(x_0^0)[x_0 - x_0^0] + \frac{1}{2}\ddot{a}(x_0^0)[x_0 - x_0^0]^2 + \dots, \quad v(r) = \frac{\dot{a}(x_0^0)}{a(x_0^0)}rc + O(2). \quad (21)$$

Since the relevant connection coefficient for radial motion as obtained from equation (2) is given by $\Gamma_{0\chi}^\chi = \frac{\dot{a}}{a}$, we see that the construction of the velocity field $v(r)$ in flat space-time depends only on the fact that this connection coefficient is non-zero. Since this is true regardless of the RW-model, interpretations of $v(r)$ depends crucially on the nature of $\Gamma_{0\chi}^\chi$. And that nature follows from sections 2 and 3 of the present paper.

A paper based on the faulty line of reasoning outlined above is [6], claiming that interpretations of spectral shifts between FOs for small distances depend on the choice of coordinate system and method of calculation. Moreover, it is argued that cosmological spectral shifts are most “naturally” interpreted as Doppler shifts in flat space-time for small distances. But as we have seen, these claims are simply incorrect. A related idea advocated in [6], is that spectral shifts between FOs can equally “naturally” be interpreted as Doppler shifts in flat space-time even for large distances. To justify this

assertion, the total spectral shift is being thought of as an accumulated effect of many small Doppler effects in flat space-time. But this logic will, of course, break down since spectral shifts between FOs cannot, in general, consistently be interpreted as Doppler shifts in flat space-time even for small distances. On the other hand, the antithesis of [6] is a paper [7] where it is (also incorrectly) argued that cosmic spectral shifts involving FOs only must “definitely” be interpreted as “gravitational” (with an exception for the Milne model). This claim is based on the specific choice of (discontinuous) scale factor $a(x^0) = 1 + \theta(x^0)$, where $\theta(x^0)$ is the Heaviside step function. It is then argued that the resulting cosmological spectral shift cannot be interpreted as a Doppler shift in flat space-time since both source and receiver are at rest when the signal is emitted or received. Moreover, it is argued that the sudden “non-local motion” occurring in this example should shed light on the interpretation of cosmological spectral shifts obtained in any “non-empty” RW-model. However, as we have seen in sections 2 and 3, a mere choice of scale factor without considering spatial geometry is not sufficient for interpretations of cosmological spectral shifts obtained in the RW-models. Besides, counterexamples of the claim that cosmological spectral shifts obtained from any non-empty RW-model must “definitely” be interpreted as “gravitational” are presented in section 3 of this paper, for situations more generally described in Definition 2 (see section 2).

There have also been earlier attempts to split up cosmic spectral shifts into “kinematic” and “gravitational” parts (for small distances). Such have been based on a Taylor expansion similar to that shown in equation (21) in combination with a Newtonian approximation to calculate the “gravitational” contribution (a second order blueshift, see, e.g., [8, 9]). However, as we have seen, using equation (21) for this purpose is misguided. Besides, since interpretations of spectral shifts in the RW-models should be based on their geometric properties only, without referring to specific dynamical laws, any use of Newtonian approximations only confuses the issue.

A recent attempt of defining said split-up in general (even for large distances) has been made in [10]. In that paper, the “recession velocity” is defined as the 3-velocity obtained by parallel-transporting the 4-velocity of the emitting FO to the observing FO along a space-like geodesic lying in a hypersurface of constant cosmic time, and then projecting the resulting 4-velocity into the local rest frame of the observing FO. This “recession velocity” then defines the “kinematic” part of the cosmic spectral shift. (But as shown in section 2, this approach does not make sense for RW-models with flat or spherical space sections since with this definition, there is no correspondence with spectral shifts in SR.) The difference between the total cosmic spectral shift and the “kinematic” spectral shift is interpreted as a “gravitational” spectral shift. It was shown that this definition of

“gravitational” spectral shift agrees with that found in [8, 9] for small distances. But while the effort made in [10] is certainly ingenious, this does not change the fact that the resulting interpretations are in general inconsistent with the geometry of the RW-models, as explained in this paper and in [1].

5 Conclusion

For several years, a debate has been going on in the scientific literature regarding the nature and interpretation of cosmological spectral shifts. This debate is primarily about theoretical models based on GR and whether or not different interpretations of cosmic spectral shifts are consistent with these models.

As a general rule, any interpretation is consistent with any theoretical model as long as no logical or mathematical inconsistencies arise. Therefore, different interpretations of the same features of any model may in principle be possible. One often hears that this is the case for the interpretation of cosmic spectral shifts. However, in this paper, we have shown that the school claiming general validity (at least for sufficiently small distances) of a “kinematic” interpretation of cosmological spectral shifts, is in error. This is so since geometric properties of the RW-models are inconsistent with such interpretations, except for the Milne model and special epochs in open RW-models. In particular, we have shown that for flat and closed RW-models, there can be no cosmic expansion without the relevant space-time curvature since otherwise, the Gauss equation would be violated. Therefore, in these models, cosmic spectral shifts must be interpreted as an effect solely due to space-time curvature. For open models, interpretations are more subtle, since here, at least part of the spectral shifts will be “kinematic”.

So, is the nature of the cosmic expansion now fully understood and all controversy settled once and for all? This is not likely, since convincing opponents of their erroneous arguments and points of view is very difficult. Besides, an alternative space-time framework exists, the so-called quasi-metric framework (QMF), where the cosmic expansion is described as new physics not covered by GR or Newtonian concepts, and its nature differs radically from its counterpart in the RW-models [3]. The QMF describes the nature of the cosmic expansion as “non-kinematic” in the sense that it is not a part of space-time’s causal structure. (Thus one may argue that the nature of the cosmic expansion in the RW-models is indeed “kinematic” in the broader sense of being part of space-time’s causal structure, without specifying any particular dynamical model.) Moreover, unlike GR, quasi-metric space-time is by construction equipped by a “preferred” global foliation into 3-dimensional, simply connected and closed spatial hypersurfaces defining “space”.

There is some resemblance to a closed RW-model since the “non-kinematic” expansion also defines extra space-time curvature via a curved connection. That is, just as for the closed RW-models, in the QMF the cosmic redshift is an effect of space-time curvature. However, a rather unique prediction of the QMF is that gravitationally bound systems should expand in general, and this prediction has observational support in the solar system [11, 12]. This means that it should be possible in principle to test the nature of the cosmic expansion by doing controlled experiments in the solar system. But based on the GR prediction that the cosmic expansion in the solar system should be far too small to be detectable, both the evidence in favour of local cosmic expansion and the possibility of doing controlled experiments to test it have been ignored so far.

As a final remark, I regret to say that if the scientific discussion regarding popular cosmology were sound, it would not have been necessary for me to write the present paper. However, in this field much low-quality and confusing material has been published by people who should know better. As a result, several incorrect arguments based on personal intuition seem to have been accepted as “mainstream”, misleading people and in particular students. Such breach of decent scholarship cannot be allowed to pass without notice.

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